Existence for the time-incremental contact problem with friction

In this talk, we focus on the incremental quasi-static contact problem with Coulomb friction in linearised elasticity (Signorini-Coulomb problem) and present optimal existence results for the most general 2D problem with arbitrary geometry and elasticity modulus tensor. The problem is reduced to a variational inequality involving a nonlinear operator which handles both elasticity and friction. This operator is proved to fall into the class of the so-called Leray-Lions operators, so that a result of Brezis can be invoked to solve the variational inequality. It turns out that one property in the definition of Leray-Lions operators is difficult to check and requires proving a new fine property of the 2D linear elastic Neumann-to-Dirichlet operator.

In the case of isotropic elasticity, either homogeneous or heterogeneous, we prove the existence of solutions to the Signorini-Coulomb problem for arbitrarily large friction coefficient. In the case of anisotropic elasticity, we exhibit an example of nonexistence of a stationary solution for large friction coefficient; then, we provide a condition on the friction coefficient under which anisotropic solutions are proved to exist. This is a joint work with Patrick Ballard.